EDC 430

As simple as a, b, c.

Challenge:

What effects do the parameters *a*, *b*, and *c* of the function $f(x) = ax^2 + bx + c$ have on the shape, orientation, and position of the graph of *f(x)?*

What we know without experimentation:

The orientation of the graph (up or down) is determined by the sign of the parameter *a* only. Positive=up; negative=down

The shape of the graph is also associated with *a* only. When *a*>1 or *a*<-1, the larger |*a*|, the "narrower" or steeper the graph. When 0<*a*<1 or -1<*a*<0, the graph is wider as *a* is closer to 0.

Rephrasing the challenge:

What happens to the position of the graph? This question can be investigated by investigating what happens to the vertex.

You have learned that parameter *c* moves the graph up and down in the plane. This notion of the effects of *a* and *c* work when we keep *b* zero. In other words when we investigate $f(x) = ax^2 + c$. The influence of *b*, and moreover of *a*, *b*, and *c* together is often not discussed in the secondary curriculum. Let's take a look and see how *a*, *b*, and *c* are related to *h* and *k,* which form the coordinates of the vertex in the vertex form, which must be equivalent by completing the square.

$$a(x-h)^2 + k =$$
$$ax^2 - 2ahx + (k + ah^2)$$

Note that therefore:

$b = -2ah \Rightarrow h = -\dfrac{b}{2a}$ This is not surprising from knowing the quadratic formula, where this represents the axis of symmetry and thus the x-coordinate of the vertex.

It also follows that:

$$c = k + ah^2 \Rightarrow k = c - ah^2 \Rightarrow$$
$$k = c - a(\frac{-b}{2a})^2 \Rightarrow$$

We now have expressed *h* and *k* in *a*, *b*, and *c*

$$k = c - \frac{b^2}{4a}$$

$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}$$

Use the Geogebra file *Quadratic Coefficients* to investigate the movement of the vertex (i.e. the graph) of *f(x)* by experimenting as follows with the provided sliders:

1. Vary *a* for several different fixed choices of *b* and *c*
2. Vary *b* for several different fixed choices of *a* and *c*
3. Vary *c* for several different fixed choices of *a* and *b*

Each time first predict what relationship you expect to find among the vertices in each of the three experiments. Establish a function that relates the vertex to the varying parameter.